

**OPTIMUM STRUCTURAL DESIGN
WITH STATIC AEROELASTIC CONSTRAINTS**

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INTRODUCTION

In this paper, the static aeroelastic performance characteristics, divergence velocity, control effectiveness and lift effectiveness are considered in obtaining an optimum weight structure. A typical swept wing structure is used with upper and lower skins, spar and rib thicknesses, and spar cap and vertical post cross-sectional areas as the design parameters. Incompressible aerodynamic strip theory is used to derive the constraint formulations, and aerodynamic load matrices. A Sequential Unconstrained Minimization Technique (SUMT) algorithm is used to optimize the wing structure to meet the desired performance constraints.

- MINIMUM WEIGHT DESIGN OF LIFTING SURFACES
- DIVERGENCE SPEED
- LIFT EFFECTIVENESS
- CONTROL EFFECTIVENESS
- FORWARD AND AFT SWEPT WINGS
- STRIP THEORY AERODYNAMICS

STATIC AEROELASTIC ANALYSIS

The equation of equilibrium is given in Equation 1. The divergence velocity is computed by setting the initial angle of attack and the flap setting to zero in Equation 1. The aerodynamic influence coefficient matrix $[A]$ is computed using the strip aerodynamics. The lift effectiveness is the ratio of flexible to rigid lift and is computed by setting the flap angle in the equilibrium equation to zero, and is given in Equation 2. The rolling of an aircraft is affected by the raising and/or lowering of a flap located on the wing. Control effectiveness is the measure of the rolling moment produced by a flexible wing to that produced by a rigid wing at an angle of attack equal to zero, and is given in Equation 3.

$$[K]\{u\} = q[A]\{u\} + q[A^r]\{\alpha^r\} + q[A^f]\{\delta\} \quad (1)$$

$$E^L = \frac{q\{h\}^T[A^r]\{\alpha^r\} + q\{h\}^T[A]\{u\}}{q\{h\}^T[A^r]\{\alpha^r\}} \quad (2)$$

$$E^C = \frac{q\{p\}^T[A^f]\{\delta\} + q\{p\}^T[A]\{u\}}{q\{p\}^T[A^f]\{\delta\}} \quad (3)$$

SENSITIVITY ANALYSIS

Mathematical optimization techniques involve computation of the search direction for finding the optimum. This involves gradients of the constraints and the objective function with respect to the design variables. In the following, gradients of the aeroelastic behavior constraints are given. Calculation of the objective function gradient with respect to the design variables is straight forward. The divergence gradients are computed using the left and right eigenvectors. The aerodynamic matrices do not vary with the design variables.

$$\frac{dq}{dx} = \frac{\{v\}^T \frac{d[K]}{dx} \{u\}}{\{v\}^T [A] \{u\}} \quad (4)$$

$$\frac{dE^L}{dx} = \frac{\{h\}^T [A] \frac{d\{u\}}{dx}}{\{h\}^T [A^r] \{\alpha^r\}} \quad (5)$$

$$\frac{dE^C}{dx} = \frac{\{p\}^T [A] \frac{d\{u\}}{dx}}{\{p\}^T [A^f] \{\beta\}} \quad (6)$$

OPTIMIZATION PROBLEM

The structural weight was minimized with limitations on divergence velocity, lift effectiveness and control effectiveness. The design variables were upper and lower skin thicknesses, cross-sectional areas of vertical posts and spar caps, and spar and rib thicknesses. Lower bounds were imposed on the design variables. The optimization problem was solved using quadratic extended interior penalty function method with Newton's method of unconstrained minimization. The computer program NEWSUMT-A was used.

Minimize the structural weight, $f(\mathbf{x})$ subject to

$$g_j(\mathbf{x}) = G_j(\mathbf{x}) - G_j \geq 0 \quad j = 1, 2, \dots, m \quad (7)$$

and

$$x_i^l \leq x_i \quad i = 1, 2, \dots, n \quad (8)$$

NUMERICAL RESULTS

The wing shown in Figure 1 is modelled with quadrilateral membrane elements for the upper and lower skins, shear panels for the ribs and webs, and rod elements for the spar caps and vertical posts. The structure consists of 66 elements, and it is made of aluminum with Young's modulus of 10.5×10^6 psi, $\nu=0.3$, and a weight density of 0.1 lb/in^3 . The wing is swept through 30 degrees representing typical forward-swept wing configurations. The wing shown has a 180 in. semispan, a constant chord of 50 in. (i.e. untapered), and a symmetric airfoil.

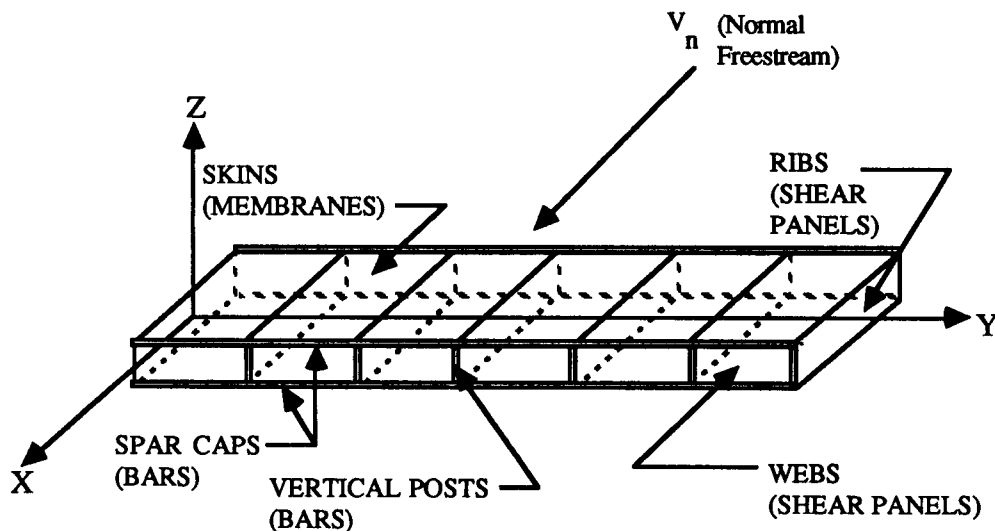


Figure 1. Built-Up Wing Configuration

Fig. 2 shows lift effectiveness and control effectiveness plotted against velocity. For this forward-swept configuration control reversal is higher than the divergence velocity. The divergence velocity is 515 ft/sec, and the control reversal (where the effectiveness goes to zero) is approximately at 1375 ft/sec. The typical nature of this plot is due to $\frac{q_R}{q_D} > 1$ as reported in Principles of Aeroelasticity by Bisplinghoff and Ashley.

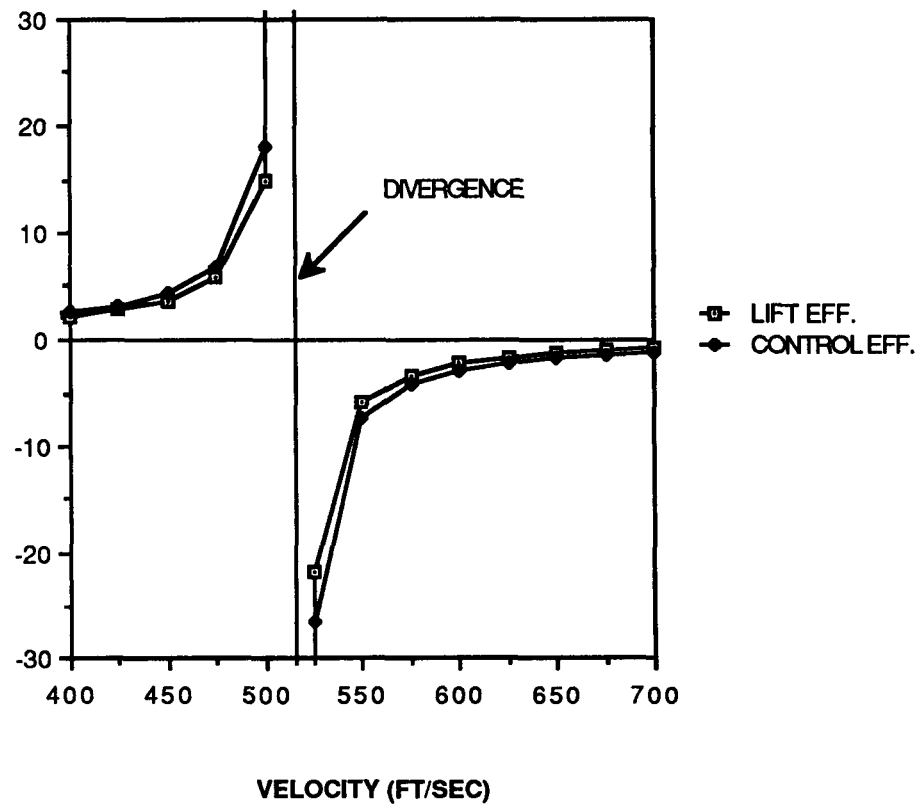


Figure 2. Control Effectiveness and Lift Effectiveness vs. Velocity

Initially the structural weight was minimized by imposing a lower limit on the divergence velocity. The divergence speed increased to 550.00 ft/sec from the reference design value of 515.06 ft/sec. A comparison of the optimum structure's divergence speed to NASTRAN analysis revealed less than a couple of percent difference at the initial and final designs. The convergence to the optimum is smooth as shown in Fig. 3.

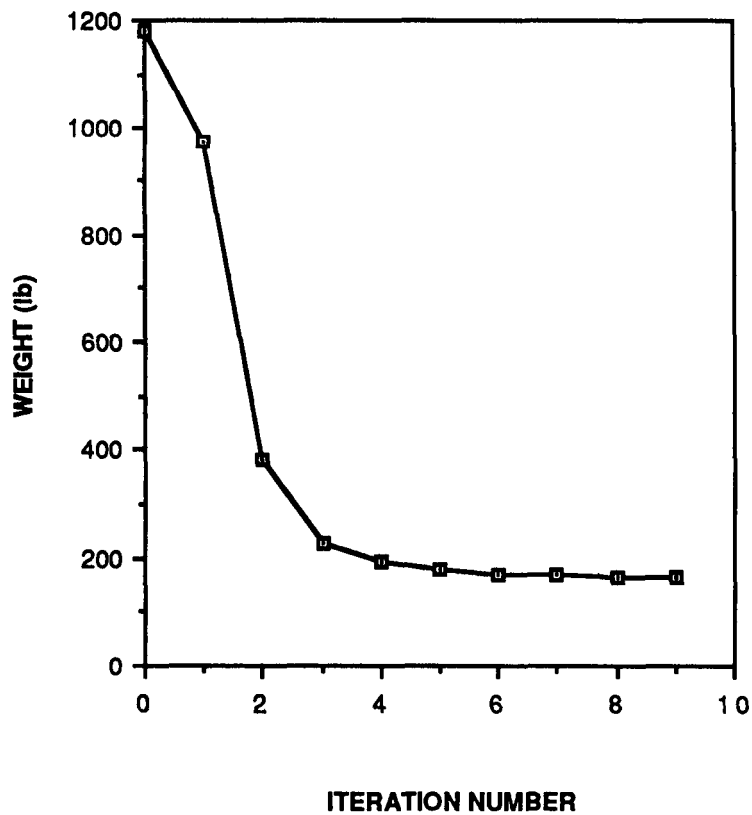


Figure 3. Design Iteration History for Divergence Constraint

The lift effectiveness and control effectiveness were calculated at the flight speed of 373.96 ft/sec, and were monitored as the divergence speed was increased. Several divergence velocities were considered for minimizing the structural weight. Fig. 4 shows the optimum weight vs. divergence velocity. The divergence speed lower limit was increased from 500 to 675 ft/sec. The optimum weight monotonically increased with the divergence velocity requirement. Lift effectiveness and control effectiveness both decreased with an increase in the divergence speed as shown in Fig. 4.

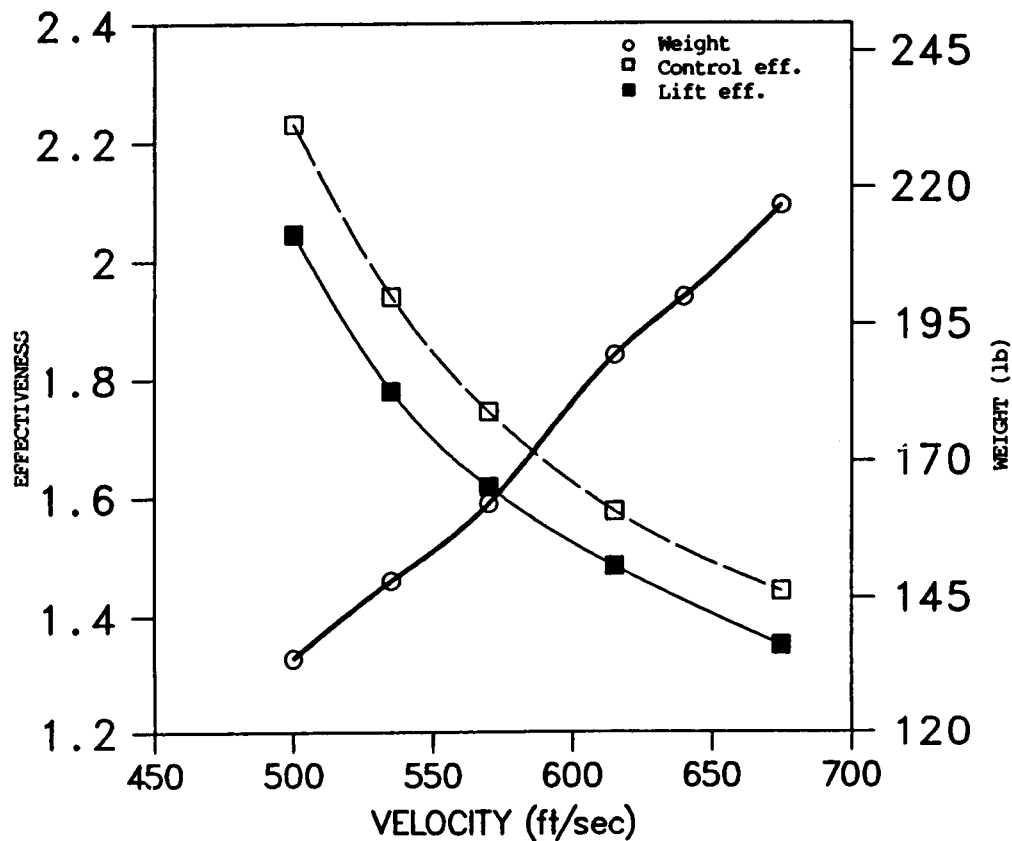


Fig. 4 Optimum Weight, Lift Effectiveness and Control Effectiveness vs. Divergence Velocity

The wing was optimized with two constraints imposed at a time to explore the opposite trend of the effectiveness values and divergence speeds with the increase of structural weight. The structure was optimized such that the divergence velocity be above 550 ft/sec and lift effectiveness above 2.0. The initial weight of the structure with all design variables set to 1.0 was 1180.80 lbs and the optimum structure had a divergence value of 560.57 ft/sec, a lift effectiveness of 2.71, and a weight of 225.46 lbs. Optimization with control effectiveness in place of lift effectiveness yielded a divergence of 550.03 ft/sec, a control effectiveness of 3.18, and a weight of 168.17 lbs.

Finally all three constraints were applied to this wing concurrently. Retaining the same constraint values as mentioned above, the structure was optimized and converged to an optimum design after six iterations. The final weight was 249.63 lbs with a divergence speed of 573.36 ft/sec, a lift effectiveness of 2.53, and a control effectiveness of 2.62. Table 1 presents the optimum performance values obtained for different combinations of constraints.

Table 1. Optimization Results

CONSTRAINTS	OPTIMUM			
	q_{DIV} (ft/sec)	E^L	E^C	Weight (lb)
$q_{DIV} \geq 550.0$ ft/sec	550.00			154.54
$q_{DIV} \geq 550.0$ ft/sec $E^L \geq 2.0$	560.60	2.71		225.46
$q_{DIV} \geq 550.0$ ft/sec $E^C \geq 2.0$	550.00		3.18	168.17
$q_{DIV} \geq 550.0$ ft/sec $E^L \geq 2.0$ $E^C \geq 2.0$	573.36	2.53	2.62	249.63

CONCLUSIONS

The divergence velocity of forward-swept wing configurations is the primary characteristic that must be improved. An increase in the structural stiffness of a wing will prevent a low divergence speed, but results in an increase of aircraft weight. Also an increase in divergence velocity affected the decrease of lift effectiveness and control effectiveness. Optimization of a wing for the three static aeroelastic constraints involves careful selection of the constraint limits. The studied wing had an initial weight of 1180.9 lbs, and lift and control effectiveness values of 1.11 and 1.12 respectively. Following the optimization process that set constraint limits on the effectiveness values of 2.0 and 550.0 ft/sec on divergence speed, the wing weighed 249.63 lbs, satisfying all constraints. The above results demonstrate the capability and feasibility of optimizing for all three constraints concurrently, rather than one at a time.

- MINIMUM WEIGHT DESIGN IS PERFORMED
FOR STATIC AEROELASTIC CONSTRAINTS
 - INCREASE IN DIVERGENCE SPEED
RESULTS IN A DECREASE OF EFFECTIVENESS
VALUES
 - CAREFUL SELECTION OF STATIC
AEROELASTIC CONSTRAINT VALUES WILL
RESULT IN SUCCESSFUL OPTIMIZATION
- PRESENT APPROACH USED NEWSUMT-A
PROGRAM
- ULTIMATE GOAL IS TO DEVELOP OPTIMALITY
TECHNIQUES

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